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Induced voltage in a 2DEG caused by a moving inhomogeneous magnetic field

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Abstract. We study theoretically the magnetic coupling between a superconducting film and a 2DEG by generalizing the theory of Meincke and that of Rammer and Shelankov, correctly taking into account the Hall effect in the 2DEG. The induced voltages are discussed in the magnetic-field regime in which the magnetoresistance is dominated by the SdH oscillations. The results obtained are found to be in good agreement with the experiment of Kruithof *et al*. We find the modulation of the magnetic field in the 2DEG is larger than previously expected.

The study of the motion of flux lines (FLs) in type-II superconductors has attracted much attention from physicists for decades [1–5]. Recent progress achieved in the high-critical-temperature superconductors have made the problem of flux motion of current interest [6–10]. In a magnetic field B, a type-II superconductor allows the field to penetrate in the form of FLs in the range of $B_{c1} < B < B_{c2}$, where B_{c1} and B_{c2} are the lower and upper critical fields of the superconductor. Each of the FLs carries a single magnetic flux quantum Φ_0, where $\Phi_0 = h/2e = 2.07 \times 10^{-15}$ T m^2. Because of the mutual interaction, in the absence of pinning, the FLs form a regular triangular lattice in the superconductor film in the case of magnetic induction perpendicular to the film. In the high-field regime, FLs are heavily overlapping due to the strong magnetic-field penetration. The magnetic field at the core of a vortex is stronger than that in between the vortices, but their difference is in general much smaller than the average field. In this regime, the magnetic field can be divided into two parts: the spatially averaged field and a small modulation.

It is also well known that when there is an electric current passing through the superconductor film, FLs will move due to the force exerted by the current [1, 2]. In contrast, the motion of FLs will in turn induce an electric field which leads to dissipation and charge redistribution in the superconductor. These ideas have long been used to study the physics of the Giaever transformer in which current-induced flux flow in the primary superconducting film induces a voltage in the magnetically coupled secondary superconducting film [3]. In a recent work, Kruithof *et al* [11] studied experimentally a related system in which the secondary superconducting film was replaced by a two-dimensional electron gas (2DEG) and found that the moving FLs in the superconducting film induced a voltage in the 2DEG as expected. They found that the induced voltage as a function of the electron density of the

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2DEG contains an oscillatory term and a background term. The oscillation is proportional to the magnetoresistance which could be interpreted qualitatively in terms of the theory of Meincke [12]. The unexplained background is correlated with the flux-flow voltage and has been ascribed to the magnetic-field-dependent Hall conductivity. This is not surprise since the theory of Meincke [12] did not take into account the Hall effect, i.e. it was assumed that electrons move in the direction of the electric field. Another available theory has been given by Rammer and Shelankov [13], who treated the conductivity tensor within the classical limit. This limit is valid only in the low-magnetic-field regime and may not give the correct answer for fields approaching $\omega_c\tau \sim 1$, where ω_c is the cyclotron frequency and τ the scattering time. Note that the Hall conductivity in the 2DEG is simply given by the expression $-ne/B + \sigma/\omega_c\tau$, where n is the areal density of the 2DEG, in a magnetic field high enough to observe the Shubnikov–de Haas (SdH) oscillation [14, 15]. This may make the analysis of the induced electric field in the 2DEG much simpler if FL motion is well controlled in the primary superconducting film. The purpose of this paper is to generalize the theory of Meincke [12] and that of Rammer and Shelankov [13] by correctly taking into account the Hall effect in the 2DEG. The induced voltages both parallel to and perpendicular to the velocity of the moving magnetic field will be discussed. To relate our results to available experiments we pay special attention to the magnetic-field regime in which the magnetoresistance is dominated by the SdH oscillations.

Although the behaviour of FL motion in a superconductor is complicated, it is true that passing an electric current (I_{DC}) through the superconducting gate can make FLs move with the velocity v_L ($v_L = v_L(I_{DC})$). The 2DEG is therefore exposed in the magnetic field of the moving FLs. To simplify the analysis, the effects of electrons in the 2DEG on the FL motion in the gate are assumed to be negligible, i.e. the FLs will move in the 2DEG with exactly the same velocity v_L as in the gate. Let the velocity v_L be a two-dimensional vector in the plane of the 2DEG and assume that the magnetic field is applied in the direction z perpendicular to the plane, then we have

$$b = B(r - v_L t) + \delta B(r - v_L t) \tag{1}$$

where $B(r - v_L t) = B\hat{z}$, $\delta B \ll B$ and $r = (x, y)$. For the sake of simplicity, we may set the averaging of δB over space and time equal to zero, i.e. $\langle \delta B \rangle = 0$. The electric field $e(r, t)$ in the 2DEG includes the field E due to the accumulation of electrons at the sides of the 2DEG and field e_i due to the moving FLs, i.e. $e(r, t) = E + e_i(r, t)$, and

$$e_i(r, t) = -v_L \times B(r - v_L t) + \delta e(r, t) \tag{2}$$

where $-v_L \times B$ and δe are the electric fields arising from the moving mean magnetic field $B(r - v_L t)$ [16, 17] and $\delta B(r - v_L t)$ respectively.

Considering the contribution of the Hall effect, the local current density in the 2DEG is formally written as [14]

$$j(r, t) = \sigma(r, t)e(r, t) - \sigma_H(r, t)\hat{z} \times e(r, t) \tag{3}$$

where σ and σ_H are the local dissipative and Hall conductivity respectively. We may represent σ and σ_H as $\sigma(r, t) = \langle\sigma\rangle + \delta\sigma(r, t)$, and $\sigma_H(r, t) = \langle\sigma_H\rangle + \delta\sigma_H(r, t)$, where $\delta\sigma$ and $\delta\sigma_H$ correspond respectively to the modulation of σ and σ_H due to the field $\delta B(r, t)$; hence we have $\langle\delta\sigma\rangle = \langle\delta\sigma_H\rangle = 0$ to the first order of δB. For the sake of brevity of mathematical representation, we use σ and σ_H to replace the mean conductance $\langle\sigma\rangle$ and $\langle\sigma_H\rangle$ in the following.

Now we can write the macroscopic electric current density $\boldsymbol{J} = \langle \boldsymbol{j}(\boldsymbol{r}, t)\rangle$ as follows:

$$\boldsymbol{J} = \sigma\boldsymbol{E} - \sigma\langle \boldsymbol{v}_{\mathrm{L}} \times \boldsymbol{B}(\boldsymbol{r} - \boldsymbol{v}_{\mathrm{L}}t)\rangle + \langle \delta\sigma(\boldsymbol{r}, t)\delta\boldsymbol{e}(\boldsymbol{r}, t)\rangle - \sigma_{\mathrm{H}}\hat{\boldsymbol{z}} \times \boldsymbol{E} + \sigma_{\mathrm{H}}\hat{\boldsymbol{z}} \times \langle \boldsymbol{v}_{\mathrm{L}} \times \boldsymbol{B}(\boldsymbol{r}, t)\rangle - \hat{\boldsymbol{z}} \times \langle \delta\sigma_{\mathrm{H}}(\boldsymbol{r}, t)\delta\boldsymbol{e}(\boldsymbol{r}, t)\rangle. \tag{4}$$

At magnetic fields in the regime of $\omega_c\tau \sim 1$, the quantized Hall effect is not resolved and SdH oscillation dominates. We can still use the semiclassical relation $\sigma_{\mathrm{H}} = -ne/B + \sigma/\omega_c\tau$ to represent Hall conductivity [14, 15], where n is the areal number density of electrons in the 2DEG. In the first-order approximation, we have

$$\delta\sigma_{\mathrm{H}}(\boldsymbol{r}, t) = \left[-\sigma_{\mathrm{H}} + \frac{B}{\omega_c\tau}\frac{\mathrm{d}\sigma}{\mathrm{d}B}\right]\frac{\delta B(\boldsymbol{r}, t)}{B}. \tag{5}$$

On the other hand, the longitudinal conductance oscillates with the variation of electron concentration n due to the formation of Landau levels (SdH oscillation). We know that the electron concentration in the field-effect transistor is proportional to the gate voltage V_g apart from a threshold V_t i.e. $n \propto (V_g - V_t)$ [14, 18]. Hence we may vary the resistivity ρ of the 2DEG for different electron density by varying the gate voltage. Adopting the experimental results of Kruithof *et al* [11], the longitudinal resistivity and the magnetic field are related by

$$\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}B} = K \sin\left[\frac{2\pi(V_g - V_t)}{\Delta V}\right] \tag{6}$$

where ΔV corresponds to the Landau-level degeneracy, and K is the experimental constant. ΔV and K vary with field B. Combining equations (5) and (6) with the relation $\rho = \sigma/(\sigma^2 + \sigma_{\mathrm{H}}^2)$, we get

$$\delta\sigma(\boldsymbol{r}, t) = \left\{c_1 \sin\left[\frac{2\pi(V_g - V_t)}{\Delta V}\right] - c_2\right\}\frac{\delta B(\boldsymbol{r}, t)}{B} \tag{7}$$

and

$$\delta\sigma_{\mathrm{H}}(\boldsymbol{r}, t) = \left\{-\sigma_{\mathrm{H}} + \frac{1}{\omega_c\tau}\left[c_1 \sin\left[\frac{2\pi(V_g - V_t)}{\Delta V}\right] - c_2\right]\right\}\frac{\delta B(\boldsymbol{r}, t)}{B} \tag{8}$$

to the first order of $\delta B/B$, where $c_1 = [(\sigma_{\mathrm{H}}^2 + \sigma^2)/(\sigma_{\mathrm{H}}^2 - \sigma^2 - 2\sigma\sigma_{\mathrm{H}}/\omega_c\tau)]\sigma BK$ and $c_2 = 2\sigma\sigma_{\mathrm{H}}^2/(\sigma_{\mathrm{H}}^2 - \sigma^2 - 2\sigma\sigma_{\mathrm{H}}/\omega_c\tau)$ are constants depending on B, and $\langle\delta\sigma\rangle = \langle\delta\sigma_{\mathrm{H}}\rangle = 0$.

To calculate the induced field $\delta\boldsymbol{e}$ we may expand it into Fourier series as $\delta\boldsymbol{e}(\boldsymbol{r}, t) = \Sigma_q \delta\boldsymbol{e}(\boldsymbol{q}) \exp \mathrm{i}\boldsymbol{q}\cdot(\boldsymbol{r} - \boldsymbol{v}_{\mathrm{L}}t)$. Combining with Maxwell equations we get the Fourier component $\delta\boldsymbol{e}(\boldsymbol{q}) = [[\boldsymbol{q}\times(\boldsymbol{v}\times\boldsymbol{q})]/q^2 - (\sigma_{\mathrm{H}}/\sigma)[\boldsymbol{q}\cdot(\hat{\boldsymbol{z}}\times\boldsymbol{v})]\boldsymbol{q}/q^2]\delta b(\boldsymbol{q})$ in the zeroth-order approximation of conductance [13, 17], where $\boldsymbol{v} = \hat{\boldsymbol{z}} \times \boldsymbol{v}_{\mathrm{L}}$ and $\delta b(\boldsymbol{q})$ is the Fourier component of $\delta B(\boldsymbol{r}, t)$. The correlation parts in equation (4) can now be determined by performing the same calculations as in [13]. Then for the triangular symmetry of FLs in the type-II superconductor, we get the macroscopic current density including all terms to the order $\langle(\delta B/B)^2\rangle$ from equation (4) as follows:

$$\begin{aligned}\boldsymbol{J} = {} & \sigma\boldsymbol{E} - \sigma_{\mathrm{H}}\hat{\boldsymbol{z}} \times \boldsymbol{E} + \sigma\boldsymbol{E}_s - \sigma_{\mathrm{H}}\hat{\boldsymbol{z}} \times \boldsymbol{E}_s \\ & + \frac{1}{2}\left\{\left[\left(-\frac{\sigma_{\mathrm{H}}}{\sigma}\right)\frac{1}{\omega_c\tau} + 1\right]\left[c_1 \sin\left[\frac{2\pi(V_g - V_t)}{\Delta V}\right] - c_2\right] + \frac{\sigma_{\mathrm{H}}^2}{\sigma}\right\}\left\langle\left(\frac{\delta B}{B}\right)^2\right\rangle\boldsymbol{E}_s \\ & + \frac{1}{2}\left\{\left[-\frac{1}{\omega_c\tau} - \left(\frac{\sigma_{\mathrm{H}}}{\sigma}\right)\right]\left[c_1 \sin\left[\frac{2\pi(V_g - V_t)}{\Delta V}\right] - c_2\right] + \sigma_{\mathrm{H}}\right\}\left\langle\left(\frac{\delta B}{B}\right)^2\right\rangle\hat{\boldsymbol{z}}\times\boldsymbol{E}_s\end{aligned} \tag{9}$$

where $\boldsymbol{E}_s = -\boldsymbol{v}_L \times \boldsymbol{B}$ [16], which is parallel to the direction of the applied current density.

In order to measure the effect of the induced electric field, it is sufficient to measure the voltage build-up across the 2DEG corresponding to the macroscopic field $\langle e \rangle = \boldsymbol{E} + \boldsymbol{E}_s$ [13, 16, 17]. For this purpose we set the macroscopic current equal to zero, i.e. $\boldsymbol{J} = \boldsymbol{0}$. Since $\boldsymbol{E}_s$ and $\hat{z} \times \boldsymbol{E}_s$ are vectors in the plane of the 2DEG and perpendicular to each other, we decompose the two-dimensional vector $\boldsymbol{E}$ along $\boldsymbol{E}_s$ and $\hat{z} \times \boldsymbol{E}_s$, i.e. $\boldsymbol{E} = c_\parallel \boldsymbol{E}_s + c_\perp \hat{z} \times \boldsymbol{E}_s$. Inserting this expression into equation (9) we can now obtain the coefficients $c_\parallel$ and $c_\perp$. In order to get definite results we assume that the superconducting gate and 2DEG have the same dimensions of width L_1 and length L_2 (in and perpendicular to the direction of the current in the gate). Then we get the induced voltage $V_{\text{ind}}^{\parallel}$ in the 2DEG measured in the direction of applied current

$$V_{\text{ind}}^{\parallel} = A_1^{\parallel} \sin\left[\frac{2\pi(V_g - V_t)}{\Delta V}\right] - A_0^{\parallel} \tag{10}$$

where $V_{\text{ind}}^{\parallel} = (c_\parallel E_s + E_s)L_1$ [16, 17] and the coefficients A_1 and A_0 are given by

$$A_1^{\parallel} = \frac{\sigma_H^2 + \sigma^2}{2(\sigma_H^2 - \sigma^2 - 2\sigma\sigma_H/\omega_c\tau)} BKV_s \left\langle \left(\frac{\delta B}{B}\right)^2 \right\rangle \tag{11}$$

$$A_0^{\parallel} = \frac{\sigma_H^2}{(\sigma_H^2 + \sigma^2 - 2\sigma_H\sigma/\omega_c\tau)} V_s \left\langle \left(\frac{\delta B}{B}\right)^2 \right\rangle \tag{12}$$

where $V_s = E_s L_1$ is the flux-flow voltage in the superconducting gate. The induced voltage in the 2DEG measured perpendicular to the applied current

$$V_{\text{ind}}^{\perp} = A_1^{\perp} \sin\left[\frac{2\pi(V_g - V_t)}{\Delta V}\right] + A_0^{\perp} \tag{13}$$

where $V_{\text{ind}}^{\perp} = c_\perp E_s L_2$ and the coefficients are given by

$$A_1^{\perp} = \frac{\sigma_H^2 + \sigma^2}{2(\sigma_H^2 - \sigma^2 - 2\sigma\sigma_H/\omega_c\tau)} (BK/\omega_c\tau) \left\langle \left(\frac{\delta B}{B}\right)^2 \right\rangle \left(\frac{L_2}{L_1}\right) V_s \tag{14}$$

$$A_0^{\perp} = \frac{(-\sigma_H)(\sigma_H^2 - \sigma^2)}{2\sigma(\sigma_H^2 - \sigma^2 - 2\sigma\sigma_H/\omega_c\tau)} \left\langle \left(\frac{\delta B}{B}\right)^2 \right\rangle \left(\frac{L_2}{L_1}\right) V_s. \tag{15}$$

It should be pointed out that the results of the induced voltages measured in the laboratory frame remain unchanged if the term $-\boldsymbol{v}_L \times \boldsymbol{B}$ is not taken into account in equation (2). However, consideration of the term has important physical consequences in that it leads to charge accumulation at two sides of the 2DEG to cancel the electric field due to the moving average magnetic field. The induced voltages (or the readings of the voltmeter) in the laboratory frame correspond to the macroscopic field $\langle e \rangle$, which contains the contribution of charge accumulation and the induction field $\boldsymbol{E}_s$ arising from the mean magnetic field [16, 17]. Experiments as done in [11] cannot distinguish whether the average magnetic field is moving or not.

It is fortunate that the induced longitudinal voltage $V_{\text{ind}}^{\parallel}$ derived here contains an oscillatory term and a negative background, in the same form as found in the experiment

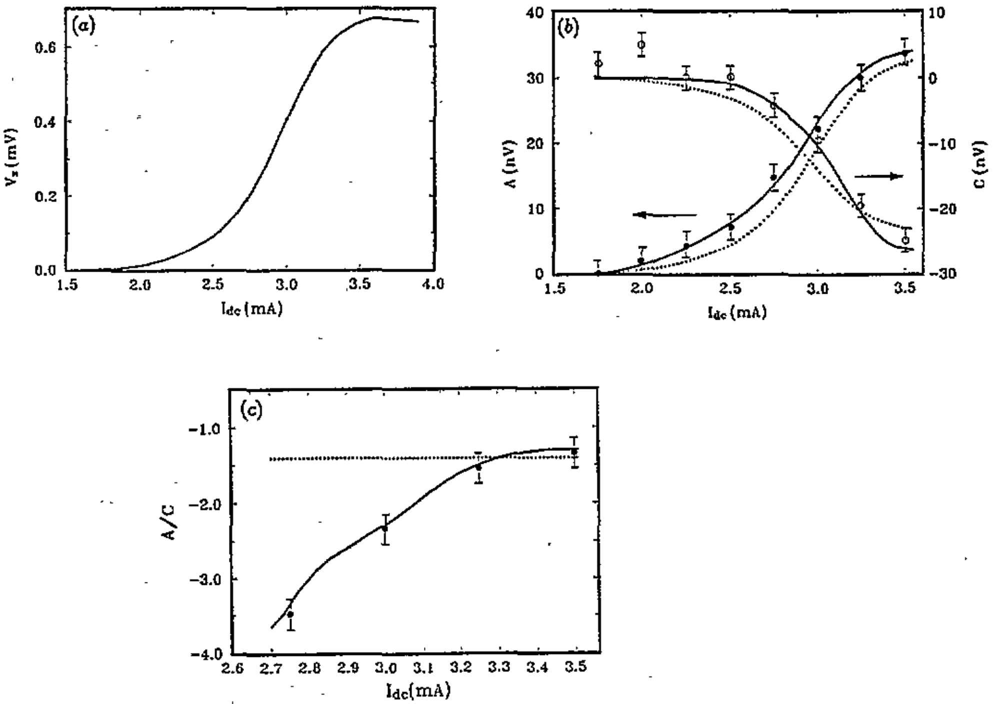

Figure 1. (*a*) The AC flux-flow voltage V_s measured with the applied DC current I_{DC} ($I_{AC} = 0.1$ mA, $T = 1.1$ K, $B = 2.1$ T) in the superconducting gate. (*b*) The experimental data (dots with error bars) and the fits (solid lines) of those data for A (left Y axis) and C (right Y axis) against I_{DC}. The dotted lines represent the theoretical results for A and C. (*c*) The experimental (solid line) and the theoretical (dotted line) results for the ratio of A to C. (The experimental data are taken from [11] point by point.)

of Kruithof *et al* [11]. To make a more detailed comparison with experiment, we change $A_1^{\parallel}$ and $A_0^{\parallel}$ into the notations used by Kruithof *et al* [11], $V_{\mathrm{ind}} = A[(1/\rho)(\mathrm{d}\rho/\mathrm{d}B)] + C$, and find that $A = A_1^{\parallel}/2$, $C = -A_0^{\parallel}$. It can be seen that A and C are proportional to the flux-flow voltage, which has been shown in experiment. It is also true that the negative background vanishes if the Hall conductivity is neglected. For convenience of comparison, the experimental data of V_s, A and C of Kruithof *et al* are shown in figure 1(*a*) and (*b*), where the dots with error bars represent the experimental data and the solid lines are the fits to those data. For a magnetic field of 2.1 T and a transport mobility of about 1 m^2 V^{-1} s^{-1} used in the experiment and taking $\langle(\delta B)^2\rangle = 1.0 \times 10^{-4}$ T, we obtain the theoretical results for A and C, which are also shown in figure 1(*b*). Considering the large errors in the experiments, this result is in qualitative agreement with the experimental values of A and C, especially the result for A. An important feature of the present work is that it predicts a ratio of A to C independent of the modulation of the magnetic field or the flux-flow voltage, provided that the coupling between the 2DEG and the superconducting gate is perfect. From (11) and (12) we get $A/C = -1.4$; this ratio is in good agreement with the experimental value as the current varies from 3.1 mA to 3.5 mA (as shown in figure 1(*c*)). As we can see from figure 1(*c*), there is a large discrepancy between our theory and the experimental data when the DC current in the superconducting gate is lower. We would like to explain this discrepancy through the following two aspects. Firstly, in this paper we have presumed that all the flux lines move with a constant velocity v_L, which depends on the Lorentz driving force exerted by the applied DC current I_{DC} in the gate. As a matter of fact, the FL motion is very complicated due to the presence of different kinds of defect in the 'dirty' superconductors. Some of the flux lines may be 'pinned' down at the defect sites and no longer free to move [19]. From the energetics point of view, the defect site is surrounded by an energy barrier which the pinned flux line must climb before it can move. The Lorentz force may effectively lower this barrier. As the Lorentz force exceeds the pinning forces, those pinned flux lines may be driven into the so called flux-flow state and undergo a viscous flow [19, 20]. Therefore, the induced voltages in the 2DEG depend on the DC current I_{DC} in the gate. Because different kinds of defect have different pinning forces, when I_{DC} is lower flux lines move in the creep-like-motion regime [19, 20], which is quite different from our presumption of uniform FL motion. Secondly, when I_{DC} is lower the experiment has large experimental errors, especially for the negative constant C, some of its experimental values are even positive (as shown in figure 1(*b*) and in [11], figure 2(*b*)). That is why the large discrepancy occurs at lower currents.

In our theory it is shown that the amplitude of the oscillation is proportional to the square of the modulation of magnetic field δB in the 2DEG, in contrast with a linear dependence anticipated by Kruithof *et al* [11]. Using the experimental value of A this would yield a magnetic field modulation δB $(= \sqrt{\langle(\delta B)^2\rangle})$ of order 10^{-2} T, which is larger than previously expected as in [11]. To explain this we would like to say the following. Firstly, in the local model [13], the conductivity tensor and the induced electric field are correlated in space and time since they originate in the same source, the modulation of the magnetic field. It is this correlation that leads to a non-vanishing induced DC electric field. Any term in order of δB becomes zero after averaging over space and time. On the other hand, we also know that the infinitely narrow magnetic-field domain, discussed by Meincke [12], is essentially different from the magnetic vortex in the type-II superconductor. Therefore, the result of [12] could not be used to explain the experiment of Kruithof *et al* as suggested in [11]. Secondly, in a magnetic field of $\omega_c\tau \sim 1$ the resistivity is dominated by the SdH oscillation and Landau quantization becomes important. The cyclotron motion of electrons may really alter the pattern of the magnetic field in the plane of the 2DEG on the scale of the cyclotron orbit [21]. If the applied modulation is ordered in the plane of the 2DEG, the alternation may be more pronounced which would lead to a larger δB than previously expected.

In order to compare quantitively our theory with the experimental data, we have taken a constant modulation of magnetic field due to FLs overlapping, i.e. the adjustable parameter $\langle(\delta B)^2\rangle = 1.0 \times 10^{-4}$ T as above. This is different from the systematic decrease of $\langle(\delta B)^2\rangle$ with increase of the DC current derived from constant A and flux-flow voltage V_s with the relation $A = V_s \delta B$, as shown in figure 3(*b*) of [11]. In this paper, we have taken the rigid pattern of FLs and considered the perfect coupling between the 2DEG and the superconducting gate for the quantitive comparison as above. In this regime, the whole arrangement of FLs would move without changing their mutual separation and the modulation $\langle(\delta B)^2\rangle$ would remain constant for fixed applied magnetic field and not change with the velocity of FLs, while in the plastic-flow regime, the FLs may move closer together when their average velocity increases [11]. This would result in the systematic increase of the overlapping of FLs with increase of velocity. Then the value of $\langle(\delta B)^2\rangle$ may decrease with increasing DC current applied in the superconducting gate. Therefore, the experimental results of Kruithof *et al* [11] have provided new information on the flux-line dynamics, which still remains to be studied in future work.

To the best of our knowledge, there has been no other available experiment with a view to measuring directly the modulation of the magnetic field in a 2DEG. Therefore we believe that it is interesting to perform experiments to measure the induced voltage, predicted by equation (13), perpendicular to the direction of the applied current in superconducting gate. As the magnetic field is increased further the 2DEG will be in the quantized Hall state. Further theoretical study is needed to reach a full understanding of the phenomenon since the results presented here are not applicable to the quantized Hall system.

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